Simulations of radiation driven islands at the density limit

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Goal: Establish the physics basis for understanding the 3D thermo-resistive effects on island dynamics

Relevant previous research efforts to augment the classical theory of resistive MHD instabilities ala FKR and FRS to include 3D thermal and/or resistive effects (partial list)

- Rebut – considered 3D resistivity with sources and sinks – but symmetric
- Suttrop – experimental evidence of asymmetric islands during disruption
- Fitzpatrick – thermal transport effects in symmetric island structures
- Hegna, Callen – ECH heating of islands, stabilizing effect
- Gates – considered implications of Rebut on density limit
- White – included asymmetric island structure and 3D resistivity quasi-linearly

This talk: show numerically in full nonlinear MHD that thermo-resistive effects and island asymmetry leads to fast exponential growth with an abrupt threshold, in agreement with analytic analysis.

Several aspects of this physics support the notion that this is an explanation of the Greenwald Density Limit.

Goal: Establish the physics basis for understanding the 3D thermo-resistive effects on island dynamics

Requirements of the numerical study:

Nonlinear full-MHD with 3D Spitzer resistivity, strong anisotropy.

Temperature profile due to radiation and Ohmic heating (im)balance inside the island (only) calculated at each time step in the simulations.

Strong anisotropy governs the outer region.

Equilibrium taken to be weakly unstable to 2/1 tearing mode in Furth, Rutherford, Selberg like profiles.

Study effects of temperature variations and island asymmetries on the order of those observed in experiments.

First step: cylindrical, allows for direct comparison with analytic theory.

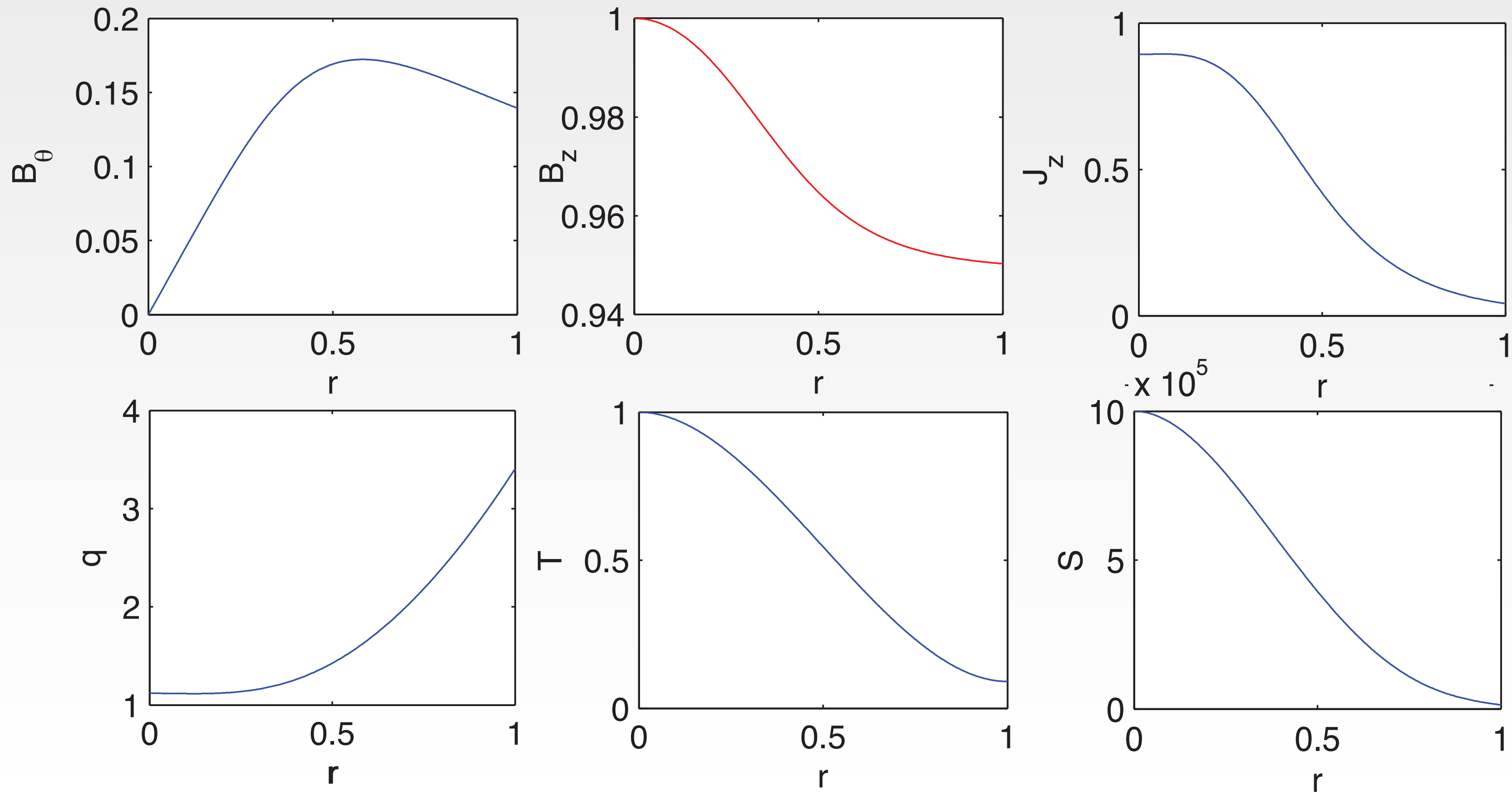
Equilibrium weakly unstable to 2/1, low β for thermal/Spitzer effect on islands

FRS J_z profile in cylinder a/R=0.5
Profiles chosen relevant to D. Gates et al, NF 12
Weakly unstable case, linear $\gamma\tau_A \sim O(10^{-3})$
Pressure force not significant, not force balance effect

$$J_z = J_0/(1+(r/r_0)^{2\nu})^{1+1/\nu}$$
$$r_0 = 0.5575$$
$$\nu = 1.53053$$

$$p = \sum_{i=0}^{3} a_i r^i$$
$$\beta = 0.005$$

With bc' s to constrain edge/core gradients



Basis for the nonlinear MHD simulations, DEBS: Nonlinear full-MHD in cylindrical geometry

DEBS advances the vector potential, velocity and pressure

D. D. Schnack et al. Comp. Phys. Comm. 43, 17 (1986).

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{V} \times \mathbf{B} - \frac{\eta}{S_0}\mathbf{J}$$

$$\frac{\partial \mathbf{V}}{\partial t} = -\mathbf{V}\cdot\nabla\mathbf{V} + \frac{1}{\rho}\mathbf{J}\times\mathbf{B} - \frac{\beta_0}{2\rho}\nabla p + \frac{\nu}{\rho}\nabla^2\mathbf{V}$$

$$\frac{dp}{dt} = -\nabla\cdot(p\mathbf{V}) - (\gamma - 1)p\nabla\cdot\mathbf{V} + \frac{1}{S_0}\nabla\cdot(\kappa_\perp\nabla_\perp T + \kappa_\parallel\nabla_\parallel T)$$

Simulation parameters and details

3D resistivity $\eta(r,\theta,z) \sim T^{3/2}$

Very low $\beta_0 = 0.005$

Thermal anisotropy (fixed) $\kappa_\parallel / \kappa_\perp = 10^5$

Semi-implicit normalized to $\tau_R = \dfrac{4\pi a^2}{c^2\eta_0}$

$$S_0 = \tau_R / \tau_A = 10^6$$

$$P = \nu / \eta_0 = 0.1$$

$$a / R = 0.5$$

where

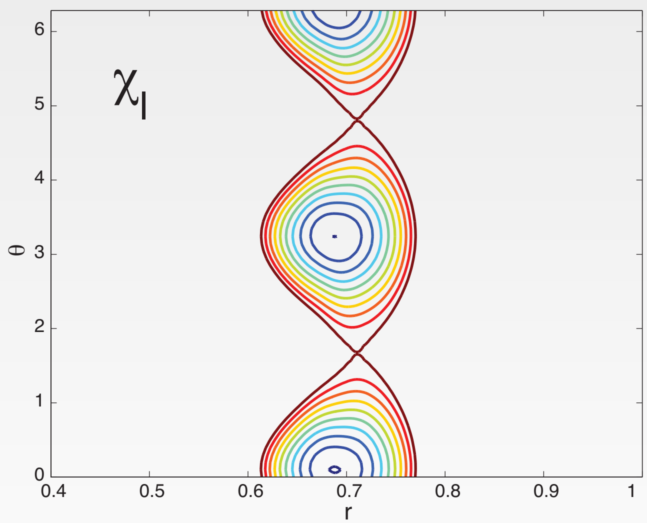
$$\nabla \times \mathbf{A} = \mathbf{B}$$

$$\nabla \times \mathbf{B} = \mathbf{J}$$

Island helical flux forms basis for temperature perturbation in nonlinear simulations

At each timestep, construct the 2/1 helical flux $\chi = mA_z + krA_\theta$

Here only the A(2/1) component is used, applicable only to 2/1 dominant cylinder



Simulations capture asymmetric deformation of flux surface, now captured in analytic analysis, critical

Inside islands, assume T is a (separate) flux function inside the islands (from thermal balance equation, previous talk)

$$0 = h\frac{\partial T}{\partial \psi} < \nabla^2 \psi > + H(T) - R(T)$$

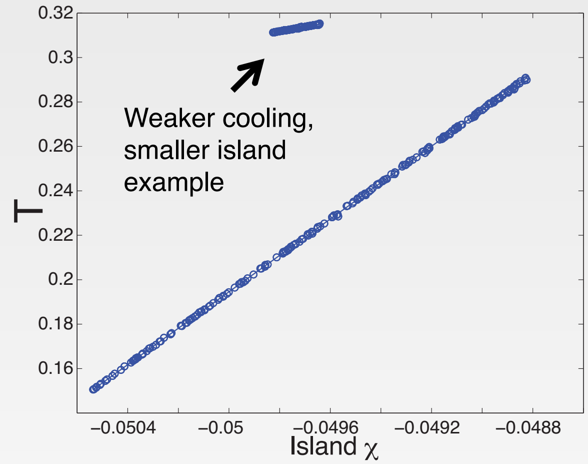
Assume that to lowest order, δT(χ) is linear in χ inside the island.

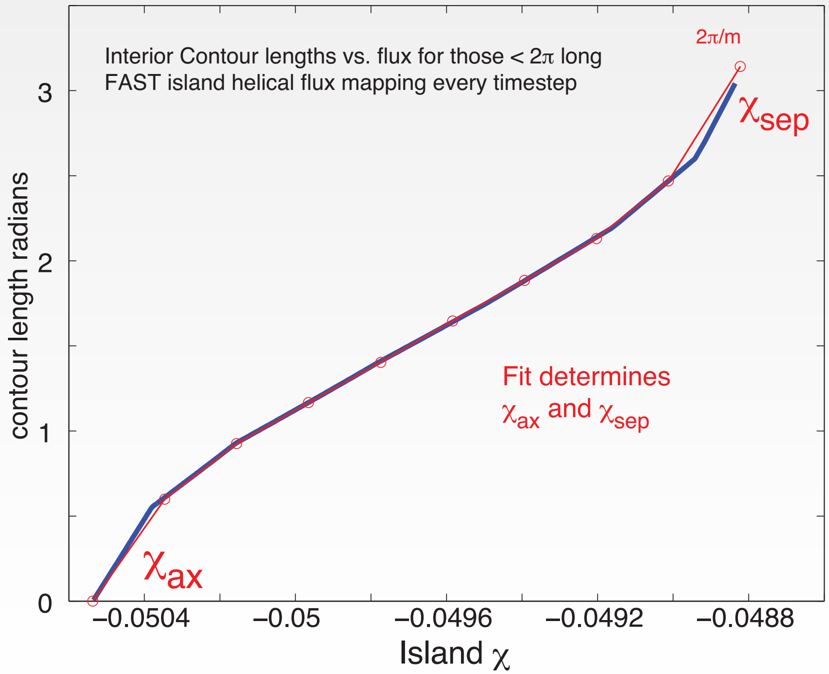
$$\delta T_I = \nabla T(\chi - \chi_{sep}) / \chi_{sep}$$

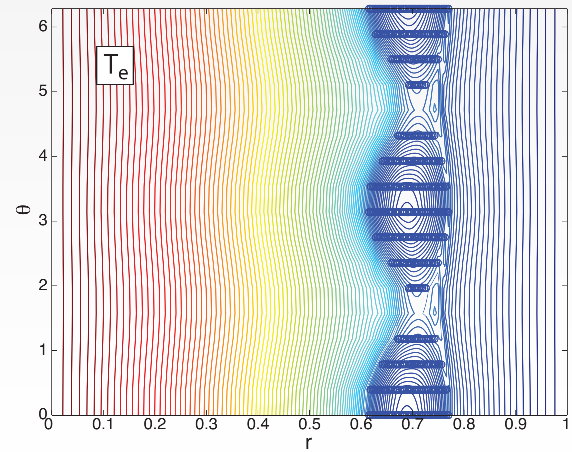
Construction of island helical flux coordinate in islands used for temperature function

The poloidal helical flux contour length, being monotonic and unique in θ, is sufficient as a coordinate basis to get χ_{ax} and χ_{sep}.
All that is needed, and fast.
Poloidal island angle coordinate could easily be constructed, if needed.

$$T_I = T_{sep} + \nabla T(\chi - \chi_{sep}) / \chi_{sep}$$







Modified Rutherford equation with radiation has exponential solution for radiation dominant growth

$$\frac{k_0}{\eta}\frac{dw}{dt} = \Delta' r_s + C_1\left(\frac{w}{w^2 + w_\chi^2}\right) + C_3\left(\frac{w^3}{w^2 + w_\chi^2}\right)$$

Radiation term

Rutherford term

Asymmetry drive (R. White, PoP 15)

- For now, ignore the bootstrap and polarization terms (consider low to moderate β_p)
- For large islands MRE then becomes:

$$\frac{k_0}{\eta}\frac{dw}{dt} = \Delta' r_s + C_3 w$$

Where: $C_3 \propto \delta P$

Exponential growth

Rebut analytic form symmetric

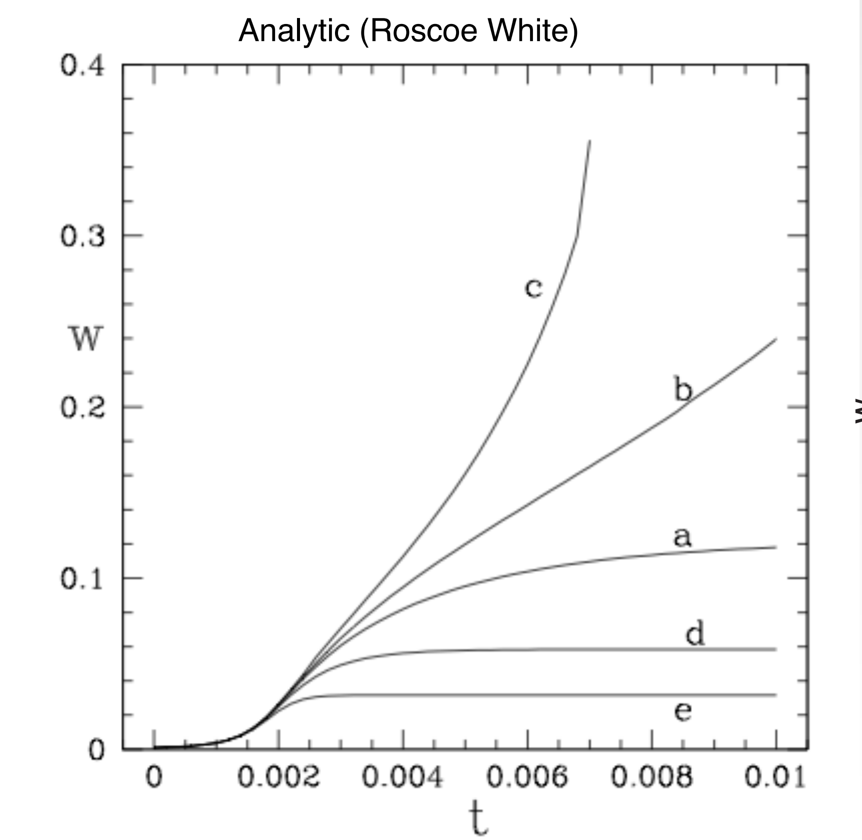
$$C_3 = 3\frac{r_s s_I}{s}\frac{\delta P}{n_e \chi_\perp T_e}$$

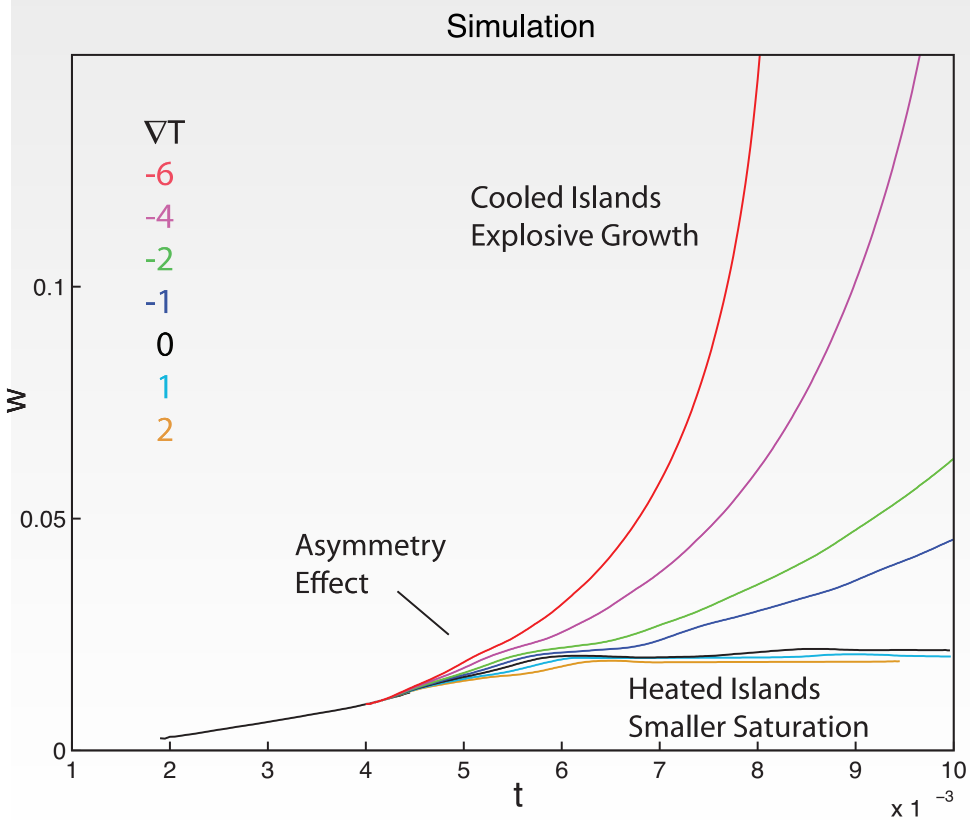
- The radiation term changes sign when $\delta P > 0$ or

$$P_{rad} = P_{island}$$

Simulations agree with reduced analysis, cooling causes exponential growth, heating causes saturation

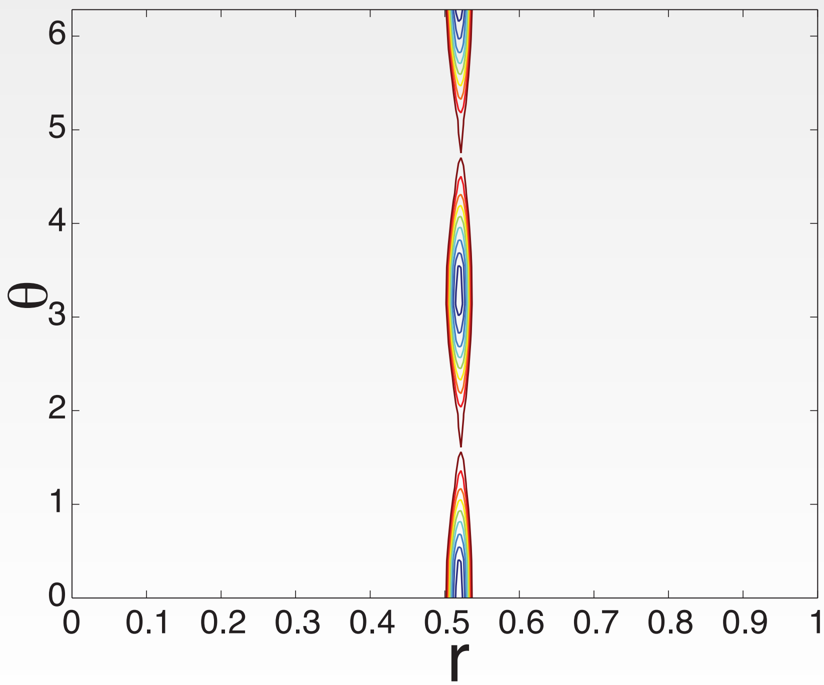
Islands with small amount of cooling eventually exponentially grow in w
Temperature perturbations are small, mostly below experimental observation
Despite tiny δT, heated islands saturate at small size ($\eta(T)$ has strong effect)

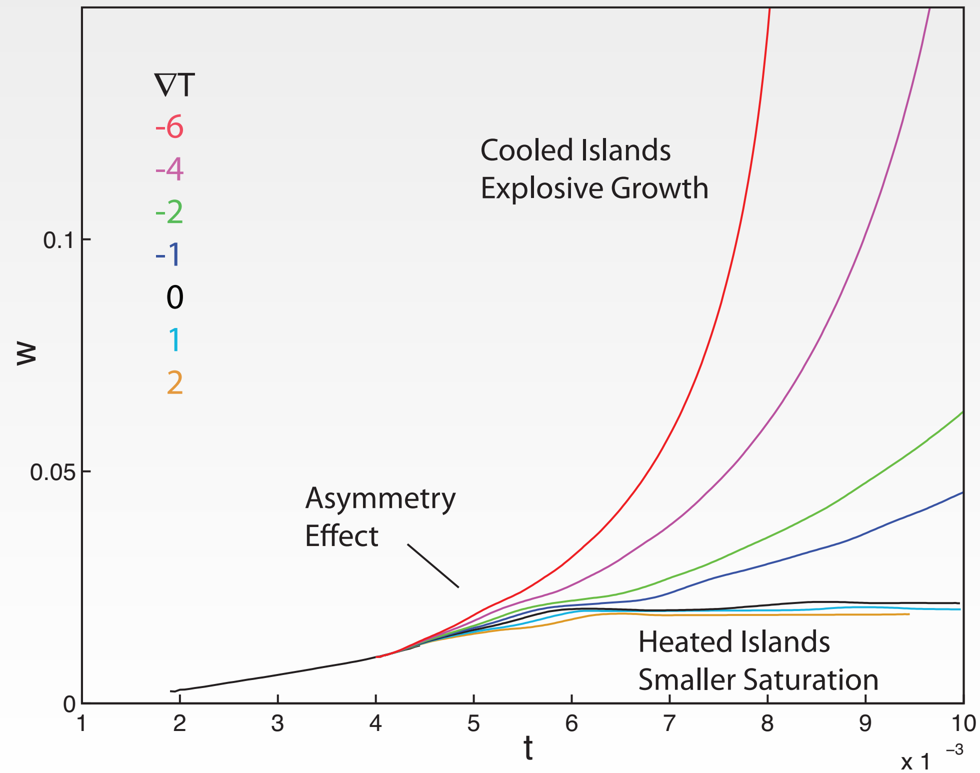




Positive temperature gradient (heated islands) are limited to small island size

Unstable mode is saturated at small island size with positive ∇T, even for significant positive Δ' (FRS peaked here with $\Delta' a \approx 4$)



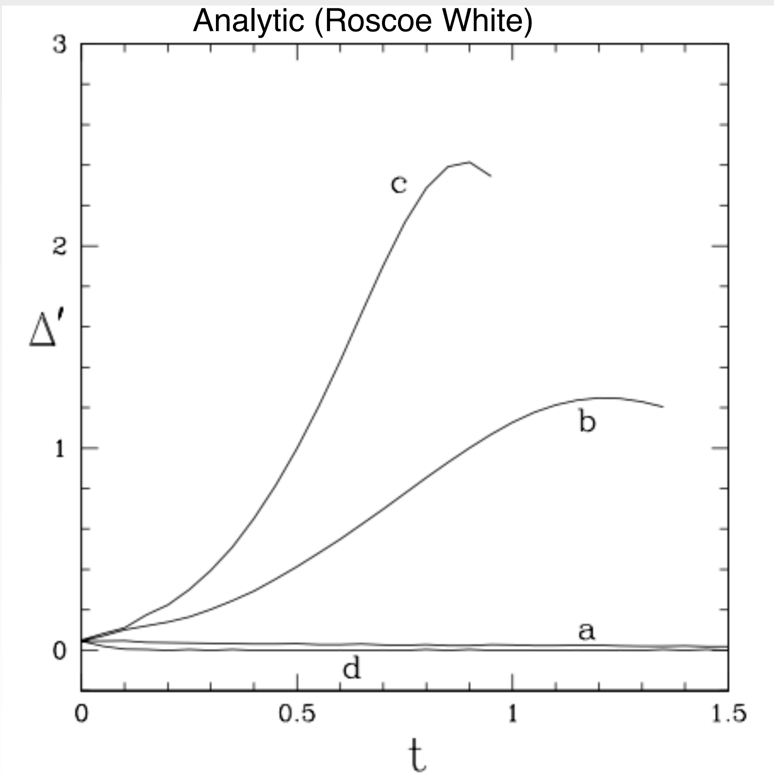


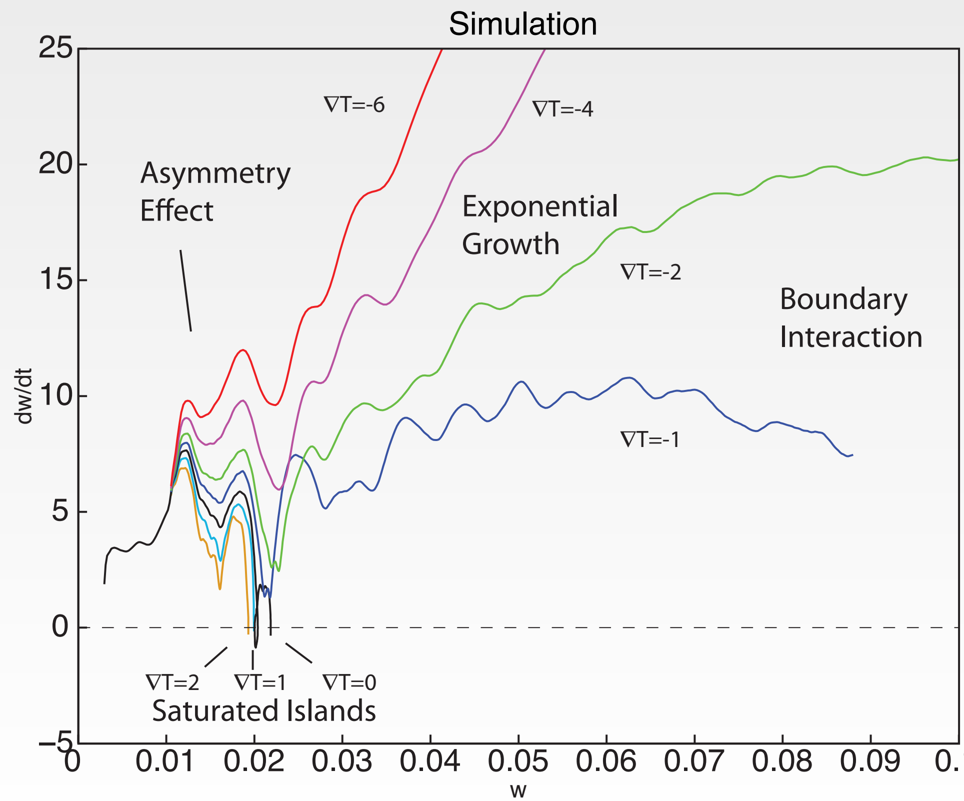
Island width growth has distinct regions, with threshold trigger to suddenly access large island

Qualitative agreement between analytic and numerical methods:

early fast growth with asymmetric island, T anisotropy and 3D η

subsequent exponential growth if cooled (and threshold surpassed)





Note: robust qualitative agreement despite significant difference between models.

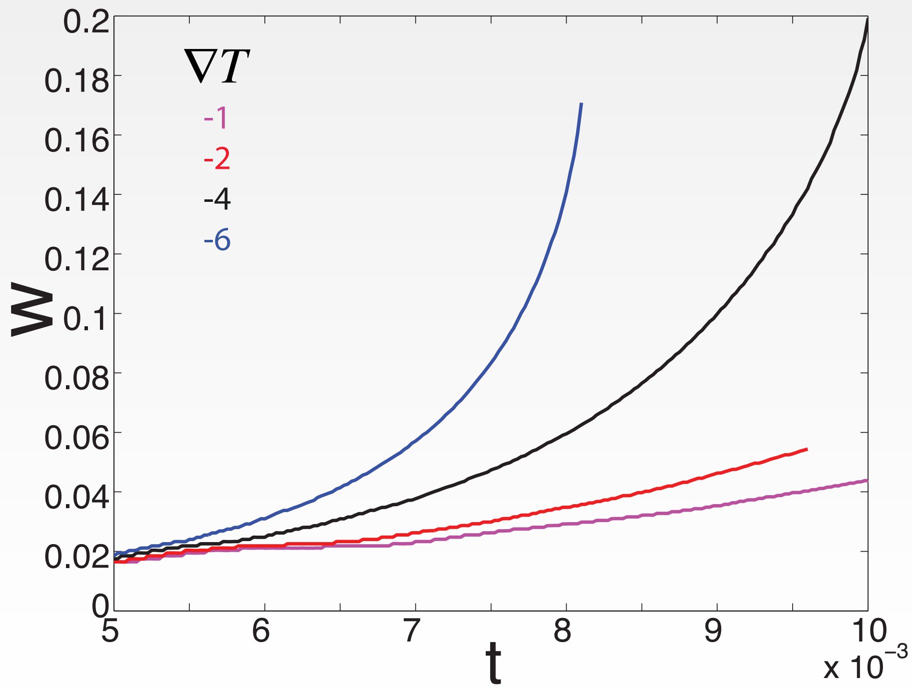
Extract the exponential factor from the simulations

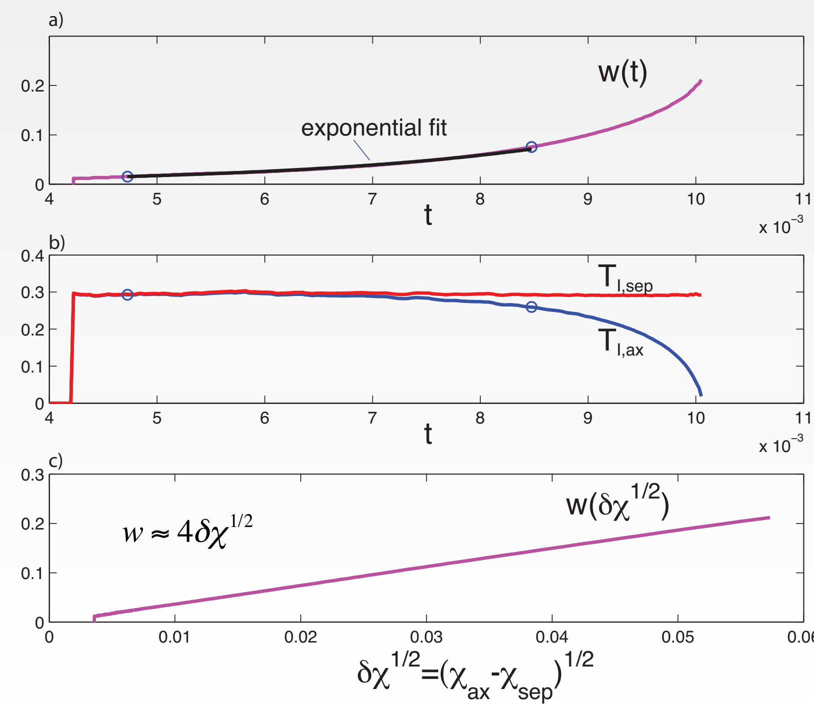
Island width closely follows an exponential in mid width range 0.03<w<0.1

Above w~0.1, interaction with the wall, m,k truncation, and sampling the equilibrium profile variation can all cause deviation from exponential

Below w~0.03, $\Delta' \sim Cw$ so deviation from exponential

Between best lower and upper limits, STD of fit and C convergence taken as conditions





Exponential growth for any negative temperature perturbation

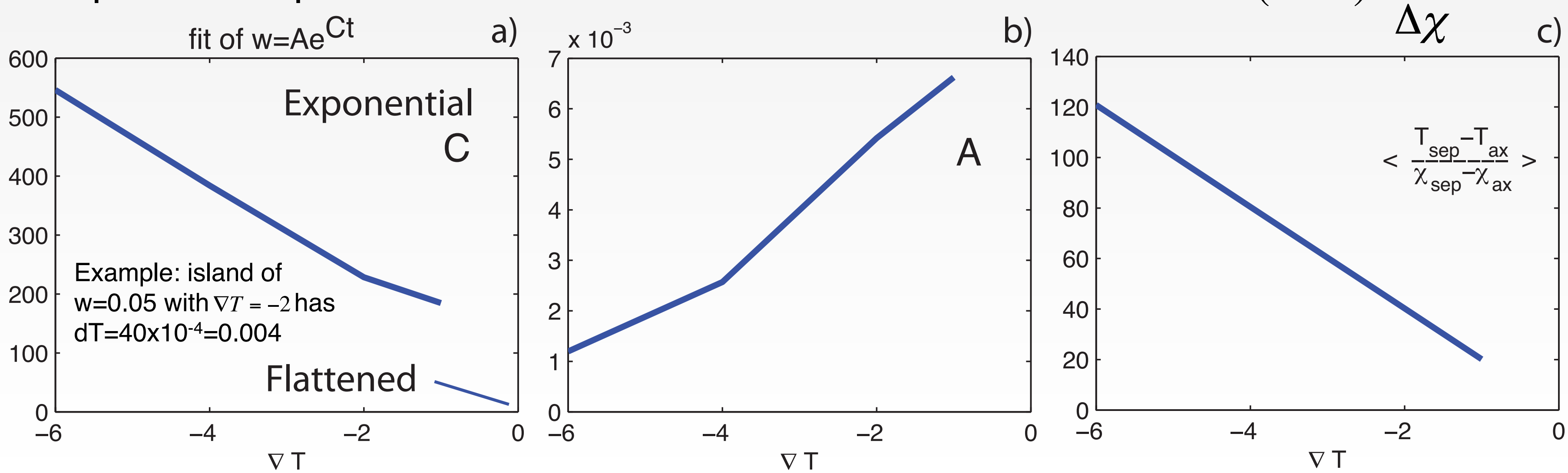
Fits project eventual exponential growth down to flattened (no cooling)

Expanding at early t $\quad Ae^{Ct} \approx A(1+Ct)$ $\qquad \dfrac{k_0}{\eta}\dfrac{dw}{dt} \approx C_3 w$

$$dw/dt \approx AC \approx \Delta' \approx O(1)$$

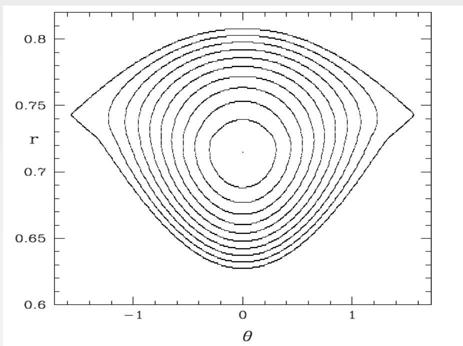
At low C the Δ' term becomes comparable to Cw

Despite fast exponentiation island's δT remains small $\qquad \delta T \approx (w/4)^2 \dfrac{\Delta T}{\Delta \chi}$

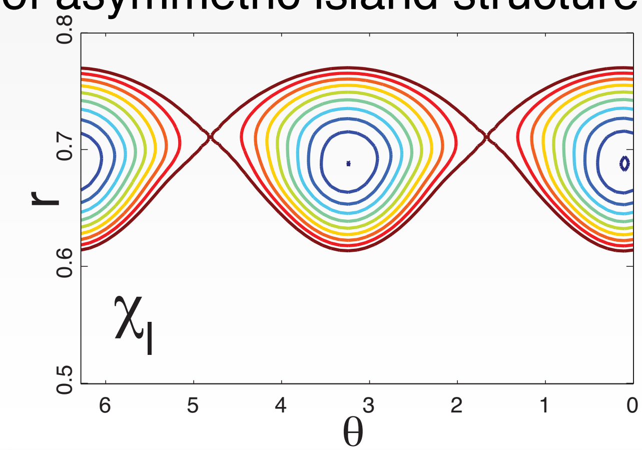


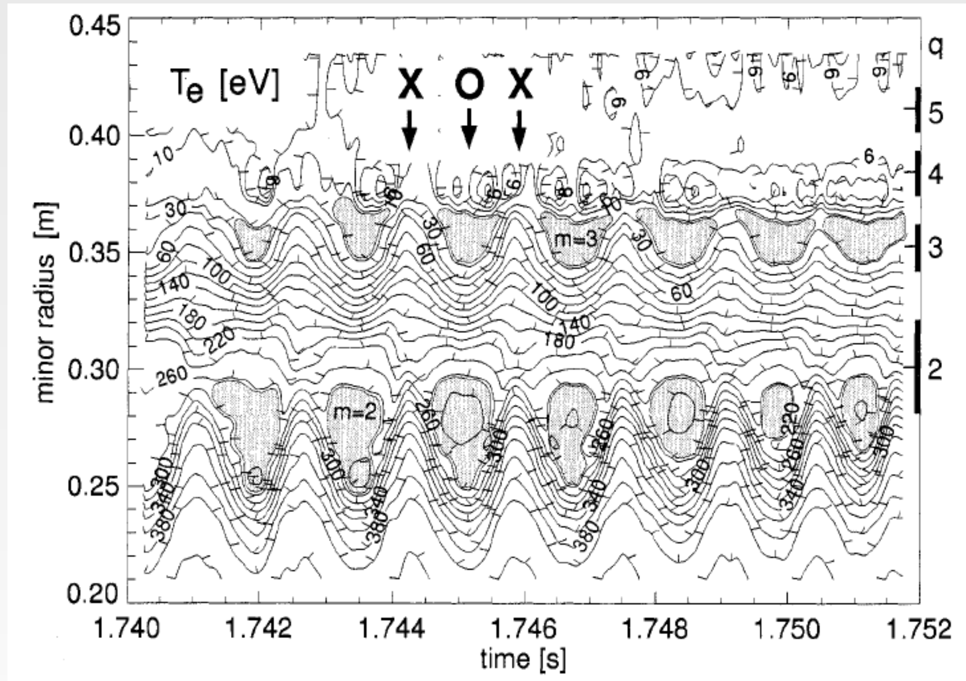
Asymmetric structure and 3D resistivity are key to capturing this physics, ignored in past

White et al: Analytic analysis of asymmetric island structure



Brennan et al: Numerical simulations of asymmetric island structure





Suttrop et al: Experimental observation (ECE) of asymmetric islands preceding a density limit disruption.

Summary and Conclusions

Numerical analysis supports analytic analysis that proper inclusion of 3D thermo-resistive effects while allowing asymmetry leads to an exponential growth mechanism with an abrupt threshold. Several aspects of this physics support the notion that this is an explanation of the Greenwald Density Limit.

Cooled islands: Exponential growth, even for tiny cooling from thermal balance

Heated islands: saturation at small size

Identified important new analytic term due to asymmetry in island structure
Verified exponential growth with full 3D resistivity and island structure in comparison with experiment

Simulations agree with asymmetric term being important at lower island width, then for cooled islands, exponentiation at larger island width

This is achieved with small temperature decreases and island asymmetries on the order of those observed in experiments

Radiation physics analyses ALSO consistent with this being an explanation of the Greenwald Density Limit (not shown here [Q. Teng, L. Delgado-Apiricio], appears in upcoming papers)

This theory provides a testable quantitative prediction of the density limit, and points to methods for exceeding the limit and controlling disruptions. Experimental proposals in progress.

Future Work

Rigorous verification of growth and saturation in cylindrical

- Simulate effect of flattening with radiation everywhere, without selecting interior region
- Need reduced perpendicular transport in island with flattening

Simulate the effect of heating and cooling in toroidal geometry

- Multiple rational surfaces with islands, in coupled nonlinear mode (not just single 2/1 cylindrical)
- Use radiation model (everywhere)
- May need reduced perpendicular thermal transport below threshold temperature gradient (inside islands)

Validate the Greenwald Density Limit against toroidal full MHD simulations

Determine effect at significant beta (playing a role in NTMs?)